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# Impulse turbine flow analysis and correlation with experimental data turbine efficiency variation with proturbances in tailpipe

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IMPULSE TURBINE FLOW ANALYSIS AND CORRELATION WITH EXPERIMENTAL DATA TURBINE EFFICIENCY VARIATION WITH PROTUBURANCES IN TAILPIPE.

BY

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IMPULSE TURBINE FLOW ANALYSIS

AND

CORRELATION WITH EXPERIMENTAL DATA

TURBINE EFFICIENCY VARIATION

WITH

PROTUBERANCES IN TAILPIPE

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# INTRODUCTION

Since the acceptance of the gas turbine as a practical means of power development innumerable improvements have been devised both experimentally and analytically which enable greater and greater amounts of power to be developed at higher and higher efficiencies from a given basic unit. Blading and nozzle design, diffuser and tailpipe configurations have all gone through alterations too numerous to mention.

Much of the steam turbine theory, in use for many years previous to the advent of the gas turbine, today plays a major role in gas turbine theory.

Just as there are a great number of different designs of gas turbine power plants and their associated components, so are there an equal number of applications, each one employing to the utmost the advantages of the particular design most suited to the case.

Thus, in the turbojet engine for aircraft, high thrust is the goal of a particular unit, whereas turbine efficiency is of secondary importance, although high losses due to friction, and inefficient blading and nozzle design may certainly result in lower net thrust.

It is the purpose of this paper to analyze the flow of combustion products through an exhaust-gas turbosupercharger. Superchargers of this type were used on

radial-engine driven aircraft during the late war as a means of providing higher available manifold pressures for flight at high altitudes and high power settings. Thus it can readily be seen that the primary object of such a turbine unit is to drive the compressor mounted on a common shaft, and that energy escaping out the tail pipe is lost energy.

Data indicative of burner or engine exhaust conditions will be used to obtain actual performance figures on the turbine. Such data will be obtained from an experimental performance analysis performed on an identical exhaust-gas turbo supercharger whose turbine was driven by combustion products from a German Jumo 004 burner using kerosene as a fuel. Fig. 1 is a schematic drawing of the test equipment, while Fig. 2 is a relatively detailed drawing of the nozzles, turbine, and tail pipe annulus, showing the arrangement of protuberances.

An attempt will be made to correlate performance figures obtained analytically with those obtained experimentally, and where disagreement occurs to offer plausible explanations and suitable remedies.

It is also the purpose of this paper to investigate the flow downstream from the turbine rotor to the extent of determining exit velocities, and velocities likely to impinge on a protuberance located in the flow path. The effect of such a flow disturbance will also

be discussed with an eye to discovering what effect, if any, such a disturbance would have on the efficiency of the turbine.

## TEST EQUIPMENT SET-UP

Fig. 1 is a schematic drawing of the test equipment used in the experimental determination of protuberance effect downstream of a turbine wheel.

Ambient air is drawn in at (T1) and its mass flow measured across a flat plate orifice at (T2). It is then compressed in a conventional turbo-supercharger (T3) installed in an Allison 1710 aircraft engine. At the entrance to the combustion chamber (T4) the pressure and temperature are measured, after which the fuel is introduced and combustion takes place in the combustion chamber (T5). At (T6) the pressure and temperature are again measured just prior to the combustion products entering the turbine nozzle box. At (T7) the pressure is assumed to be atmospheric. This, together with the values at (T6), plus the fuel and air mass flows, are used to determine the ideal turbine power.

The turbine wheel and compressor are mounted on a common shaft as indicated, hence the total power output of the turbine is used to drive the compressor. The latter draws in ambient air at (C1) after which its pressure drop across a flat plate orifice is measured at (C2). A throttling valve controls the mass flow entering the compressor, thereby permitting altitude simulation. Compressor entrance pressure and temperature are measured at (C3) as are the exit conditions at (C4).

Thus it is possible with such an independent arrangement of test equipment to compare turbine and compressor powers and by suitable calculations to obtain turbine efficiencies.

Inasmuch as the total power output of the turbine is utilized to drive the compressor, any obstruction or similar device placed in the tailpipe (8) which might act to reduce the efficiency of the turbine would be felt at any given power setting as a loss in compressor power.

Obstructions in the form of flat plates and cylindrical struts were placed in the annular space immediately downstream from the turbine wheel. Their lateral distance from the turbine wheel was varied, approaching during one set of readings to within 7/16 inch of the turbine blades. The maximum area occupied by the obstructions at the above minimum lateral distance was one third the area of the annular tail pipe and was accomplished by four flat plates located at 90° intervals and each occupying a 30° sector of the annular area. To mount the obstructions closer laterally to the turbine wheel, or to block off a greater portion of the annular space was considered inadvisable both from an experimental and practical standpoint, since the aim of the project was to investigate the pressure influence effect on turbine performance created by protuberances placed in a relatively high velocity flow, rather than to investigate choking in the tail pipe.

## Greek Notation.

$\rho$ - Density

$\varphi$ - Nozzle coefficient

$\psi$ - Blading coefficient

$\omega$ - Angular velocity

$\Phi$ - Torque

$\alpha$ - Angle between rotor plane and absolute velocity

$\beta$ - Angle between rotor plane and relative velocity

$\eta$ - Efficiency

## Subscripts.

1 - Blade entrance and nozzle exit

2 - Blade exit and compressor entrance

3 - Compressor exit and tailpipe annulus

4 - Nozzle entrance

5 - Turbine exit

a - Axial

a/f - Air plus fuel

c - Compressor

i - Isentropic

n - Nozzle

o - Ideal

p - Pitch

r - Ratio

st - Stage

t - Rotor

u - Tangential

Analysis of Energy Transfer Between Fluid and Rotor.

The analysis presented here is based on References (1), (2), and (3).

The theory employs the following simplifying assumptions:

(a) The flow through the rotor is steady and uniform over the entrance and exit cross sections of the flow passage.

(b) The rotor, to which the blades which form the flow passages are attached, rotates with a uniform angular velocity.

(c) There are no losses due to fluid by-passing the flow passages formed by the blading.

(d) There is no friction loss due to the sides of the rotor being in contact with the fluid.

(e) The fluid completely fills the flow passages in the rotor.

Actual machines, of course, never behave in accordance with the above assumptions. However, since losses external to the rotor are neglected, as is the fluid which by-passes the blading, the theory is concerned only with the fluid which actually flows through the blading.

From Fig. 3 it is seen that the axial components of relative and absolute velocities are as follows:

$$Ca = Wa = C \sin \alpha = W \sin \beta \qquad (1)$$

In general, a relative velocity is given by the vector equation,

$$W = C - u \tag{2}$$

and its tangential component $W_u$ by

$$W_u = W \cos \beta \tag{3}$$

The tangential component of the entrance absolute velocity $C_1$ is

$$C_{1u} = W_{1u} + u_1 = C_1 \cos\alpha_1 = W_1 \cos\beta_1 + u_1 \tag{4}$$

The tangential component of the exit absolute velocity $C_2$ is

$$C_{2u} = W_{2u} + u_2 = C_2 \cos\alpha_2 = W_2 \cos\beta_2 + u_2 \tag{5}$$

The relationship between the velocity vectors C, W, and u is obtained by using the law of cosines.

Hence $$W^2 = C^2 + u^2 - 2\,u\,C \cos\alpha \tag{6}$$

But $$C_u = C \cos\alpha$$

Thus $$W^2 = C^2 + u^2 - 2uC_u \tag{7}$$

or $$C^2 + u^2 - W^2 = 2uC_u \tag{7a}$$

For the entrance and exit velocity triangles of Fig. 3 equations similar to (7a) may be written.

Thus, $$C_1^2 + u_1^2 - W_1^2 = 2u_1C_{1u} \tag{8a}$$

$$C_2^2 + u_2^2 - W_2^2 = 2u_2C_{2u} \tag{8b}$$

Subtracting equation (8b) from (8a)

$$(u_1C_1u - u_2C_2u) = \frac{1}{2}\Big[(C_1^2 - C_2^2) + (u_1^2 - u_2^2)$$

$$+ (W_2^2 - W_1^2)\Big] \qquad (9)$$

Remembering the assumption of a constant weight rate of fluid flow, equation (9) takes on the form of an energy equation. Multiplying equation (9) by 1/g represents an energy transfer relationship for 1 lb. of fluid.

As a result of the energy transfer there is a torque interaction between the fluid and rotor resulting in a tangential force acting on the rotor. The moment of this force around the axis of rotation is the torque due to the above interaction. The product of the torque and the angular velocity of the rotor gives the rate of energy transfer. The magnitude of the torque is equal to the rate of change in the angular momentum of the fluid between the entrance and exit sections. This momentum is decreased in a turbine.

The angular momentum theorem applied to a flowing fluid states that the time rate of change of the resultant angular momentum of a system of discrete particles in a given direction is equal to the moment ~~amount~~ of the external forces acting in the same direction.

Denoting the energy transfer in foot-pounds per pound of fluid by L; angular momentum by MV; torque by $\Phi$ ; and energy transfer from fluid to rotor by

subscript (t), we have from equation (9) after substituting $\omega R$ for u, and removing $\omega$.

$$\bar{\Phi}_t = M_1 - M_2 = \frac{1}{g}(R_1C_{1u} - R_2C_{2u}) \tag{10}$$

The energy transfer per pound of fluid is then

$$L_t = \bar{\Phi}_t \omega = \frac{\omega}{g}(R_1C_{1u} - R_2C_{2u}) \tag{11}$$

Equation (11) shows that the energy transfer depends only upon the tangential components $C_{1u}$ and $C_{2u}$, and the tangential velocities $u_1$ and $u_2$. Any change in the axial components $C_{1a} = W_{1a}$, and $C_{2a} = W_{2a}$ is effective only in producing axial thrust on the rotor. However, as we shall see later, they play an important role in determining the carry-over velocity at the turbine exit, and may thus influence stage conditions through their interaction with disturbances which may occur downstream.

The magnitudes of $C_{1u}$ and $C_{2u}$ are controlled by the angles with which the fluid enters and leaves the flow passage.

With regard to the losses to be expected in the flow passage such as from friction, heat transfer, shock, etc., equation (11) is independent of these losses since it is concerned only with the inlet and exit conditions. However, where losses do occur they will affect the exit conditions and so be accounted for.

A product of the form $RC_u$ in equation (11) is termed the WHIRL of the fluid and the velocity change

$\Delta C_u = C_{1u} - C_{2u}$ is called the whirl velocity. Thus it is seen that the energy transfer depends directly upon the change in whirl produced between entrance and exit of the flow passage.

In Axial-flow machines $R_1 = R_2 = R$ equals the pitch radius, and $u_1 = u_2 = u$, so that equation (11) without the centrifugal effect becomes

$$L_t = \frac{u}{g}(C_{1u} - C_{2u}) = \frac{u}{g}\Delta Cu \tag{12}$$

From equations (9) and (12)

$$L_t = \frac{1}{2g}\left[(C_1^2 - C_2^2) + (u_1^2 - u_2^2) + (W_2^2 - W_1^2)\right] \quad \text{ft lb/lb} \tag{13}$$

Since $u_1 = u_2$ in an axial-flow machine

$$L_t = \frac{1}{2g}\left[(C_1^2 - C_2^2) + (W_2^2 - W_1^2)\right] = \frac{u}{g}\Delta Cu \tag{14}$$

where $\Delta C_u = C_{1u} - C_{2u}$. In terms of the angles $\alpha_1$, $\beta_1$, and $\beta_r$,

$$L_t = \frac{u}{g}(C_1 \cos\alpha_1 + W_2 \cos\beta_2 - u) \tag{14a}$$

or $$L_t = \frac{u}{g}(W_1 \cos\beta_1 + u + W_2 \cos\beta_2 - u)$$

$$= \frac{u}{g}(W_1 \cos\beta_1 + W_2 \cos\beta_2) \tag{15}$$

From the above equations it can readily be seen that in order to obtain the maximum energy transfer, the exit velocity $C_2$, and the nozzle angle $\alpha_1$ should be as small as possible.

In considering the flow through the turbine blades it is convenient to write the energy equation for the rotating blade passage in terms of the enthalpy change. Thus,

$$0 = J \int_1^2 dh + \frac{1}{2g} (W_2^2 - W_1^2) \tag{16}$$

Although the flow is adiabatic it is accompanied by friction. Let $dE_f$ equal the work expended in overcoming friction, then

$$J \int_1^2 dh = - \int_1^2 vdp + \int_1^2 dE_f \tag{17}$$

For this process the available energy is given by

$$H = - \int_1^2 \frac{v}{J} dp \tag{18}$$

For convenience let

$$JH = \frac{Co^2}{2g} \tag{19}$$

Combining equations (17), (18), and (19) gives

$$H = \frac{Co^2}{2g} = - \int_1^2 \frac{v}{J} dp = \frac{W_2^2 - W_1^2}{2gJ} + \int_1^2 \frac{dE_f}{J} \tag{20}$$

Considering the losses in the blade passage to consist of those associated with the energy of the fluid at the entrance to the passage, and those occurring within the passage itself, we can express these losses as kinetic energies associated with the fluid at entrance and exit sections of the passage.

Let $\rho_1 \frac{W_1^2}{2g}$ = energy loss at passage entrance.

$$\rho_2 \frac{W_2^2}{2g} = \text{energy loss in passage itself.}$$

Since $\int_1^2 dE_f$ is the total energy loss in the flow passage,

$$\int_1^2 dE_f = \rho_1 \frac{W_1^2}{2g} + \rho_2 \frac{W_2^2}{2g}$$

Then from equation (20)

$$\frac{Co^2}{2g} = JH = \frac{W_2^2 - W_1^2}{2gJ} + \rho_1 \frac{W_1^2}{2g} + \rho_2 \frac{W_2^2}{2g} \tag{21}$$

or,

$$\frac{Co^2}{2g} = \frac{W_2^2}{2g}(1 + \rho_2) + \frac{W_1^2}{2g}(\rho_1 - 1) \tag{22}$$

Then,

$$W_2^2 = \frac{1}{1+\rho_2}\left[Co^2 + (1 - \rho_1)\, W_1^2\right] \tag{23}$$

Introducing blading coefficients defined by

$$\psi_1^2 = 1 - \rho_1$$

$$\psi_2^2 = \frac{1}{1 + \rho_2}$$

We have finally

$$W_2 = \psi_2 \left(Co^2 + \psi_1^2 W_1^2\right)^{\frac{1}{2}} \tag{24}$$

However, in a pure impulse blade the enthalpy change from entrance to exit is zero, consequently $Co^2 = 0$ since we are assuming that the blading considered here is of the pure impulse type.

Therefore,

$$W_2 = \psi_2 \psi_1 W_1 = \psi W_1 \tag{25}$$

Fig. 5 shows the relationship between blading coefficient and blade angle. Since the blading of the rotor in question has entrance and exit angles of approximately 45°, a value for $\psi = .\overline{95}^2$ is considered reasonably accurate.

## Analytical Procedure.

From known values at station 4 (entrance to nozzle box) we have $P_{T4}$, $T_{T4}$, and wt. flow in lbs/sec. Knowing the nozzle exit area we can divide the wt. flow by the nozzle exit area and obtain wt.flow/sec.ft.$^2$ nozzle exit area. Then from Keenan and Kaye Gas Tables, pp 215-216,

$$W_n = \frac{P_4(\overline{m})^{\frac{1}{2}}}{(T_4)^{\frac{1}{2}}} \left(\frac{2g}{\overline{R}} \frac{n}{n-1}\right)^{\frac{1}{2}} \frac{1}{r^n} \left(1 - r^{\frac{n-1}{n}}\right)^{\frac{1}{2}} \qquad (1a)$$

where $\overline{m}$ = molecular wt. at (4)

$\overline{R}$ = gas const. = 1545

$n = K \approx 1.3$

Then using tables 24 and 29 we can solve for r = pressure ratio across nozzle. If we assume that the nozzle entrance velocity is zero, then the entrance total pressure = entrance static pressure = $P_{T4}$, and $P_1 = r \times P_{T4}$.

Assuming isentropic expansion in the nozzle the ideal discharge velocity is

$$C_1{}^1 = \left(2g\frac{k}{k-1} RT_{T4} \quad 1-\left(\frac{P_1}{P_{T4}}\right)^{\frac{k-1}{k}} + C_4{}^2\right)^{\frac{1}{2}} \qquad (2a)$$

(assume $C_4 = 0$)

From reference (1) p. 102,

$$C_1 = \varphi_1 C_1' \quad \text{where } \varphi_1 = 0.95 \quad \text{(p. 433 ref. 1)}$$

We are now able to draw the entrance velocity triangle for the blading passage using either a measured value of $\alpha_1$, or a calculated value as in ref. (1) p. 443, eq. 92, and the above calculated value of $C_1$, and the known value of u. We can thus measure $\beta_1$ from the triangle and check it against that measured from the blade. There should be only one value of u at which the two are equal--the turbine design point--all other points showing a discrepancy which will affect the blading efficiency.

To draw the exit velocity triangle we must make one of two assumptions, either that the exit absolute velocity is in an axial direction, or that the exit relative velocity is in a direction tangent to the blade trailing edge. An assumption is necessary inasmuch as we have no means for determining the actual flow conditions at the blade exit.

The first assumption would be incorrect since it is only true for the operating design point of the turbine. The second assumption we can be reasonably certain will be very close to the actual condition.

Hence if we assume $\beta_2$ equal to the actual physical blade exit angle we may now draw the exit velocity triangle, since we have values for $\beta_2$, u, and, as previously shown, eq. (25), we can calculate $W_2$.

As we have seen from equation (14a)

$$Lt = \frac{u}{g} (C_1 \cos\alpha_1 + W_2 \cos\beta_2 - u)$$

It is convenient for purposes of calculation to express the above equation in a slightly different form as follows

From equation (25) $W_2 = \psi W_1$

Expressing $W_1$ in terms of $\alpha_1$, u, and $\beta_1$,

$$W_1 = \frac{C_1 \cos\alpha_1 - u}{\cos\beta_1} \tag{3a}$$

Substituting the above values of $W_2$ and $W_1$ in equation (14a) gives the following:

$$Lt = \frac{u}{g} \left[ C_1 \cos\alpha_1 - u + \psi(C_1 \cos\alpha_1 - u) \frac{\cos\beta_2}{\cos\beta_1} \right]$$

or,

$$Lt = \frac{u}{g} (C_1 \cos\alpha_1 - u)(1 + \psi \frac{\cos\beta_2}{\cos\beta_1}) \tag{4a}$$

Introducing the velocity ratio $V_1 = u/C_1$

$$Lt = \frac{u^2}{g} (\frac{\cos\alpha_1}{V_1} - 1)(1 + \psi \frac{\cos\beta_2}{\cos\beta_1}) \tag{5a}$$

The above equation represents the energy transferred to the rotor by the fluid which passes through the blades. If the fluid flows through the blading at the rate of G lb/sec the power developed at the periphery of the turbine wheel is

$$HP_{St} = \frac{G}{g} u^2 (\frac{\cos\alpha_1}{V_1} - 1)(1 + \psi \frac{\cos\beta_2}{\cos\beta_1}) \tag{6a}$$

However, the above power is not all transferred to the rotor shaft. Some power is consumed in overcoming the rotation loss of the stage. The latter comprises the losses due to disk friction and blade windage loss, due to operation with partial admission. The rotation loss in horsepower can be calculated from Kerr's formula

$$h\,p_r = \left\{ k_1 D + n k_2 (1 - \epsilon)\, l^{1.5} \right\} \left(\frac{u}{100}\right)^3 \frac{Dp}{v}$$

An additional loss of original available energy results from the fact that the blade exit velocity $C_2$ cannot be used to transfer energy to the rotor but must be lost down the tailpipe.

Ordinarily in calculating stage efficiency the above losses would be included to reduce the amount of power transferred to the rotor that actually is produced at the shaft. However, for our purposes of analysis we are interested only in the energy transmitted by the fluid to the rotor, since that was the only consideration in the experimental turbine efficiency analysis.

We are, therefore, interested only in conditions of total pressure and temperature at the entrance and exit of the stage. Such conditions automatically include any carry-over velocities which may occur at either of the two stations.

We may then represent the stage horsepower as follows:

$$HP\,S_t = \frac{G}{g} u^2 \left(\frac{\cos\alpha_1}{V_1} - 1\right)\left(1 + \psi \frac{\cos\beta_2}{\cos\beta_1}\right)$$

and the turbine efficiency by

$$\eta_b = \frac{HP_{st}}{1.415\,(h_{T4} - h_{21})G} \qquad (7a)$$

Immediately downstream of the turbine rotor the tailpipe annular area is considerably greater than the turbine exit area. Hence we have a sudden expansion of the exhaust gases and an accompanying pressure loss. However, in this analysis we are assuming incompressible flow throughout, with the result that there is no change in density anywhere in the flow, and hence the only change in the axial component of the turbine exit velocity is that due to the sudden increase in flow area.

Accordingly since ${}_2 = {}_3 =$

then $A_2V_2 = A_3V_3$ where $V_2 = C_2 \sin {}_2$ (8a)

Solving for $V_3$ we obtain the velocity that exists in the annular section of the tailpipe immediately downstream from the turbine rotor.

SAMPLE CALCULATION

Rotor Speed, 10,000 R.P.M.

From Table 1.

$$W_{a/f} = W_a + W_f$$

$$= \frac{1}{60}(70 + 83/60) = 1.189 \text{ lb/sec}$$

Nozzle Exit Area = 9.2 in.$^2$

$$\therefore W_n = \frac{W_{a/f}}{9.2/144} = \frac{1.189 \times 144}{9.2} = 18.6 \text{ lbs/sec.ft.}^2$$

nozzle exit area

From Ref. (6)

$$W_n = \frac{P_{T4}\,(m)^{\frac{1}{2}}}{(T_{T4})^{\frac{1}{2}}} \left(\frac{2g}{R}\,\frac{n}{n-1}\right)^{\frac{1}{2}} \frac{1}{r^{n}} \left(1 - r^{\frac{n-1}{n}}\right)^{\frac{1}{2}}$$

$$18.6 = \frac{.49115}{(1772)^{\frac{1}{2}}} \cdot 34.48\,(28.97)^{\frac{1}{2}} \cdot .144 \left(\frac{64.4}{1545} \cdot \frac{1.3}{.3}\right)^{\frac{1}{2}}$$

$$\times \frac{1}{r^{n}} \left(1 - r^{\frac{n-1}{n}}\right)^{\frac{1}{2}}$$

$$\therefore .1404 = \frac{1}{r^{n}} \left(1 - r^{\frac{n-1}{n}}\right)^{\frac{1}{2}}$$

From Table 29 (ref. 6)

$$r = .90 + \frac{25}{134} \cdot .02 = .9037$$

Then $P_{T4} \cdot r = P_1 = .9037 \cdot 34.48 = 31.2$ in Hg

$$C_1{}^1 = \left(2g\,\frac{k}{k-1}\,R\,T_{T4}\left[1 - \left(\frac{P_{T5}}{P_{T4}}\right)^{\frac{k-1}{k}}\right]\right)^{\frac{1}{2}}$$

$$= \left(\frac{64.4}{.3} \cdot 1.3 \cdot 53.3 \cdot 1772\left[1 - (.9037)^{.231}\right]\right)^{\frac{1}{2}}$$

$$= 782.5 \text{ ft/sec}$$

Then - $C_1 = \varphi_1 C_1{}^1 = .95 \times 782.5 = 744.0$ ft/sec

$$u = \frac{rpm}{60} \times \pi \times Dp = \frac{10,000}{60} \times \pi \times \frac{11}{12} = 480 \text{ ft/sec}$$

$$u/C_1 = \frac{480}{744} = .645$$

From Fig. 4

$$\alpha_1 = 18^\circ 54'$$

$$\cos\alpha_1 = .9459$$

From Velocity Triangle Fig. 7a.

$$\beta_1 = 47^\circ 05'$$

$$\cos\beta_1 = .68093$$

$$W_1 = 330 \text{ ft/sec}$$

Then $W_2 = \overline{.95^2} \times 330 = 298$ ft/sec

From Exit Velocity Triangle Fig. 7b.

$$C_2 = 340 \text{ ft/sec}$$

$$\sin\alpha_2 = .61704$$

$$C_2\sin\alpha_2 = 210 \text{ ft/sec}$$

Now: $$L_t = \frac{u^2}{g}\left(\frac{\cos\alpha_1}{V_1} - 1\right)\left(1 + \varphi\frac{\cos\beta_2}{\cos\beta_1}\right)$$

$$= \frac{480^2}{32.2}\left(\frac{.9459}{.645} - 1\right)\left(1 + \frac{.95^2 \times .707}{.68093}\right)$$

$$= 6450 \text{ ft.lb./lb.}$$

$$HP_t = \frac{G \times Lt}{550} = \frac{1.189 \times 6450}{550} = 13.94 \text{ h.p.}$$

$HP_i = 1.415 \times G\,(h_{T4} - h_{21})$ where $h_{21}$ is the enthalpy corresponding to barometric pressure.

From Ref. 6, Table I.

$h_{r4} = 442.1$ BTU/lb.

$p_{r4} = 107.15$

$$\therefore p_{r2i} = 107.15 \times \frac{29.03}{34.48} = 90.3, \text{ and } h_{2i} = 421.78 \text{ BTU/lb}$$

Then $HP_i = 1.415 \times 1.189\ (442.1 - 421.78) = 34.2$ h.p.

$$\therefore \eta_b = \frac{13.94}{34.2} = 40.75\ \%$$

Assuming $P_{2i} = P_2 = P_1 = 31.2$ in Hg

Then $p'_{r2i} = 107.15 \times \frac{31.2}{34.48} = 97.0$

and $h'_{2i} = 430.15$

$\therefore HP'_i = 1.415 \times 1.189\ (442.1 - 430.15) = 20.2$ h.p.

and $\eta_{st} = \frac{13.94}{20.2} = 69.0\ \%$

Due to sudden expansion downstream from rotor

$\rho_2 A_2 V_2 = \rho_3 A_3 V_3$, and since $\rho_2 = \rho_3$ (incompressible flow)

$$V_3 = \frac{A_2 V_2}{A_3} = \frac{11}{28.75} \times C_2 \sin\alpha_2 = \frac{11.0}{28.75} \times 210$$

$$= \underline{\underline{80.4}} \text{ ft/sec.}$$

## RESULTS AND DISCUSSION

As a result of the analysis developed, and applied to actual operating conditions, several discrepancies in the experimental test data necessitate discussion. In fact, the discrepancies are apparently so great that a correlation of experimental and analytical data becomes an impossibility.

Table 1 is a summary of experimental test results made with a clear tail pipe, i.e., with no protuberances installed. Table 2 gives the values of ideal horsepower available across the turbine, and horsepower transmitted to the turbine rotor, as calculated analytically using the same flow conditions at turbine nozzle entrance. A rather obvious impossibility is at once apparent in the discrepancy between the values of horsepower transmitted to the turbine rotor as listed in Table 2, and the values of compressor power as listed in Table 1. If compared alone these values would indicate a turbine efficiency of well over 100%, a physical impossibility. However, when we consider that these figures do not account for losses due to turbine disk friction, windage, fluid leakage, bearing friction, and compressor windage and bearing friction, the discrepancy becomes even more glaring, inasmuch as all such losses tend to reduce the amount of power transmitted to the turbine rotor

that eventually is measured as the compressor power tabulated in Table 1.

A brief explanation of experimental procedure may serve to show cause for disbelief of the test results.

The equation employed to solve for turbine horsepower is as follows:

$$HP_t = Cp(T_{T4}-T_{T5})\eta_t \, 778/550 = 1.415 \, CpT_{T4}\left[1-\left(\frac{P_{T5}}{P_{T4}}\right)^{\frac{k-1}{k}}\right]\eta_t$$

Where $p_{t4}$ and $T_{t4}$ represent nozzle box total entrance conditions. $P_{T5}$, on the other hand, was assumed to be equal to atmospheric pressure, whereas it has been shown analytically that this is not the case (page 15). $P_{T5}$ represents the total pressure at the turbine exit. From the analytical results obtained at a turbine rotor speed of 10,000 rpm. the static pressure at the turbine exit was 31.2 in.Hg., and the total pressure would be even higher by an amount equal to $\frac{(C_2 \sin\alpha_2)^2}{2g}\rho_2$ lbs/ft$^2$.

Such a rise in $p_{t5}$ would cause a corresponding increase in the experimental values of turbine efficiency. According to reference (5) the maximum turbine efficiency obtained from a similar turbine was approximately 65%. Thus the values as obtained from the investigation under question were obviously too high for the most part to begin with, and with the added increase pointed out above, would be still further out of line. However, by accounting for the actual total turbine exit pressure, the values of efficiency so corrected would more closely

follow the trend obtained in ref. (5). That is, the correction noted above would increase the values of efficiency at higher rpm's more than those corresponding to a low rpm, since the exit total pressure increases with $p_{t4}$, as seen in Table 2.

In attempting to arrive at a plausible cause for the discrepancies noted above, we can only assume that the nozzle entrance data as recorded was wrong, or that the compressor inlet and exit conditions as recorded were in error, or both.

If we assume $T_{t4}$ to be actually two to three hundred degrees higher than as recorded we immediately raise the pressure ratio across the nozzles (equation (1a)) page 15, thus increasing considerably the value of $C'_1$, eq. (2a) page 15. This, in turn, reduces the velocity ratio $u/_{c1}$ making that parameter more in accord with the corresponding values of ref. (5), and finally increases the values representing the power transmitted to the rotor by the fluid, eq. (6a) page 17. However, the stage efficiency denoted by $\eta_{st}$ in Table 2 is changed only slightly, and the theoretically available energy, $HP'_i$ , Table 2 is not increased sufficiently to begin to compensate for the high values of compressor power, Table 1. Therefore, since it is unreasonable to assume that $T_{t4}$ is low by more than two to three hundred degrees, it is logical to believe that the values of compressor power are in error by a considerable amount. The latter power was computed as follows:

$$HP_c = Wc\ cp\ (T_{t3} - T_{t2})\ 1.415$$

Inasmuch as $T_{t3}$ and $T_{t2}$ were measured by single thermocouples located at exit and entrance stations of the compressor respectively, we might logically expect one or both to be in error, since the usual procedure in such a case is to read an average value of a bank of thermocouples located at such critical stations.

Another possibility of error might exist in the measurement of the mass flow entering the burner. Too low a value of weight flow would cause both the values of horsepower transmitted to the rotor, and ideal available turbine horsepower to be low. In the same sense the value of mass flow through the compressor might easily be too high.

Thus we can only conclude that a very real discrepancy does exist, that it may be a combination of several errors or of only one or two. The need for accuracy in the measurement of temperatures, pressures, and weight flow of fluid is clear, for without that accuracy any amount of data is almost sure to be meaningless except where it is possible to demonstrate a trend.

Fortunately such was more or less the case in the investigation under discussion.

We were interested in the effect on turbine efficiency that the placing of protuberances in the tail pipe might have. The comprehensive data obtained experimentally which has been shown quite conclusively to have

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

been in error fundamentally in the foregoing analysis, was yet extremely consistent regardless of the size, configuration, or positioning with respect to the turbine rotor of the protuberances.

Table 2 gives the values of the axial components of exit velocities from the turbine blading as well as the reduction in those velocities due to the sudden expansion in area immediately downstream of the turbine rotor. These latter velocities then, will impinge on any protuberance placed downstream.

The efficiencies ( $\eta_b$ ) as listed in Table 2 are completely arbitrary with regard to definition. They merely serve as an illustration of a trend in efficiency of energy transfer from the fluid to the rotor. Actually the efficiency of a single process may be defined in several different ways by as many authors. This writer is of the opinion that an efficiency definition to be acceptable must not only be clearly defined, but must also be used consistently throughout any given analysis or report.

In the case at hand we are interested in analyzing the flow process with a view toward correlating said analysis with that performed experimentally as described previously.

Thus we are assuming that the ideal energy available for doing work on the turbine is the isentropic enthalpy difference between the flow conditions at

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] of [illegible]

[illegible]

nozzle entrance and tailpipe exit (atmospheric). Actually, the energy available for doing work in an impulse turbine is that contained in the velocity of the fluid at the exit from the nozzle which may be represented on the enthalpy entropy chart as the isentropic enthalpy drop between the pressure at nozzle inlet and that at the nozzle exit, since the pressure remains constant through the blading.

Thus if this conception of available energy were used in the calculations a considerable rise in efficiency would be noted. For comparison purposes this latter calculation was made, and appears in Table 2 as $\eta_{st}$.

In considering the losses occasioned by the introduction of protuberances in the flow downstream of the turbine rotor, we are interested not in the loss of tailpipe residual energy, but rather in the effect such a loss might have on the efficiency of the turbine. It is known, for example, that a loss of total pressure is suffered in the flow of a fluid about an object in the flow path. When the object has the form of a cylinder the pressure loss is proportional to the sum of the squares of the radial and circumferential components of velocity of the fluid relative to the cylinder. Since the degree of flow deflection decreases nearly in proportion to the square of the radial distance from the cylinder, the loss is confined within rather narrow

limits, but appears in an analysis as a loss in total pressure differential between entrance and exit conditions of any particular flow system.

Such a pressure differential exists in even a constant area duct if we assume incompressible flow with friction present.

However, as long as the total pressure beyond the exit of a duct remains below that at the exit section of the duct, the flow will continue to be accommodated regardless of internal losses without any change in entrance conditions. In the case of the velocities existing in the tailpipe annulus as listed in Table 2 under ($C'_2$), it is considered that their values are so low as to cause only minor frictional losses in the flow.

The rise in static pressure on the upstream side of any protuberance situated in the flow due to the impingement of such flow velocities would be so small, and would extend such a minute distance upstream, that to cause any change in the nozzle discharge static pressure, the protuberances would have to be so close to the turbine blades as to be impractical.

Even considering such a possibility the slight rise in static pressure due even to the existence of the flat plate sectors described earlier would have a negligible effect on the turbine performance. Only at very high turbine exit velocities would such an effect be appreciable, and even then the protuberance

[illegible]

[illegible] tions of any particular flow system.

[illegible]

Accordingly, as long as the total pressure beyond the wall of a duct [illegible] at the wall surface of the duct, the flow will continue to be [illegible]

[illegible]

[illegible]

installation would have to be made at very close range to the turbine rotor.

Hence we may reasonably state that where tailpipe losses are of no concern, the existence downstream of protuberances such as heat exchangers or the like will have little or no effect on the turbine performance as long as relatively low velocities are involved, and providing choking or excessive losses to a condition below ambient do not occur

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- TABLE I -

| RPM | Wa lbs/min | Wf lbs/hr | $P_{T4}$ in. Hg | $T_{T4}$ °R | $P_{T5}$ in. Hg | $Hp_c$ hp | $\eta_t$ % |
|---|---|---|---|---|---|---|---|
| 10000 | 70.0 | 83.0 | 34.48 | 1272 | 29.08 | 27.1 | 75 |
| 12000 | 86.3 | 96.0 | 36.88 | | | 38.2 | 67 |
| 14000 | 102.6 | 104.0 | 38.68 | | | 53.0 | 60 |
| 16000 | 116.5 | 113.0 | 41.58 | | | 66.1 | 57 |
| 18000 | 132.5 | 126.0 | 45.58 | | | 88.5 | 54 |
| 20000 | 148.7 | 134.0 | 48.88 | | | 113.0 | 55 |

Above data obtained from experimental test.

$P_{T5}$ (turbine exit total pressure) assumed equal to atmospheric pressure.

$Hp_c$ obtained from -

$$Hp_c = W_{ca}\, c_p (T_{T3} - T_{T2})\ 778/550$$

(where $T_{T3}$ and $T_{T2}$ are Compressor exit and entrance temperatures)

$Hp_t$ obtained from -

$$Hp_t = W_{a+f}\, c_p\, T_{T4} \left[1 - \left(\frac{P_{T5}}{P_{T4}}\right)^{\frac{k-1}{k}}\right] \eta_t\ 778/550$$

Then -

$$\eta_t = \frac{Hp_c}{Hp_t}$$

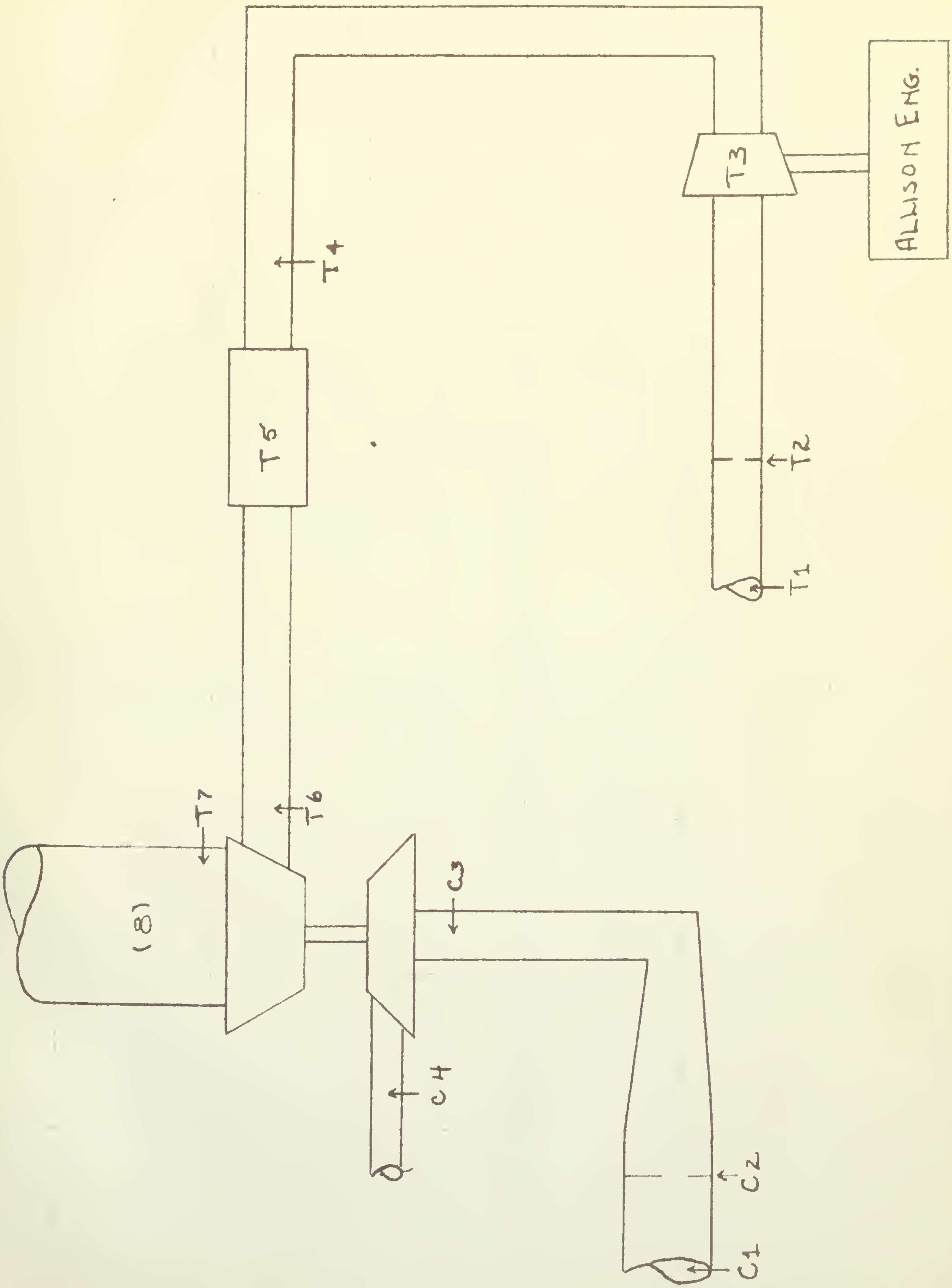
# - TABLE II -

| RPM | $W_{a+t}$ lbs/sec | $W_n$ lbs/sec/ft² | r | $P_1$ in.Hg | $C_1$ ft/sec | $U/C_1$ | $h_D$ Ft-lb/lb | $HP_c$ hp | $HP_i$ hp | $\eta_D$ % | $HP'_i$ hp | $\eta_{st}$ % | Casing ft/sec | $C'_r$ ft/sec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | 1.189 | 18.6 | .9037 | 31.2 | 744 | .6450 | 6450 | 13.94 | 34.2 | 40.25 | 20.2 | 69.0 | 210.0 | 80.4 |
| 12000 | 1.465 | 22.9 | .8653 | 31.9 | 888 | .6485 | 9230 | 24.60 | 57.7 | 42.7 | 35.4 | 69.6 | 248.0 | 95.0 |
| 14000 | 1.739 | 27.25 | .8149 | 31.5 | 1048 | .641 | 13000 | 41.00 | 82.0 | 50.0 | 59.3 | 69.2 | 300.0 | 114.9 |
| 16000 | 1.975 | 30.9 | .7845 | 32.6 | 1140 | .674 | 14710 | 52.90 | 116.1 | 45.6 | 80.1 | 66.0 | 310.0 | 118.8 |
| 18000 | 2.243 | 35.1 | .7600 | 34.6 | 1210 | .713 | 15730 | 64.25 | 163.0 | 39.42 | 102.0 | 63.0 | 335.0 | 128.2 |
| 20000 | 2.515 | 39.4 | .7200 | 35.2 | 1320 | .727 | 18220 | 83.3 | 210.5 | 39.50 | 131.5 | 63.2 | 347.0 | 133.0 |

Summary of DATA CALCULATED Analytically.

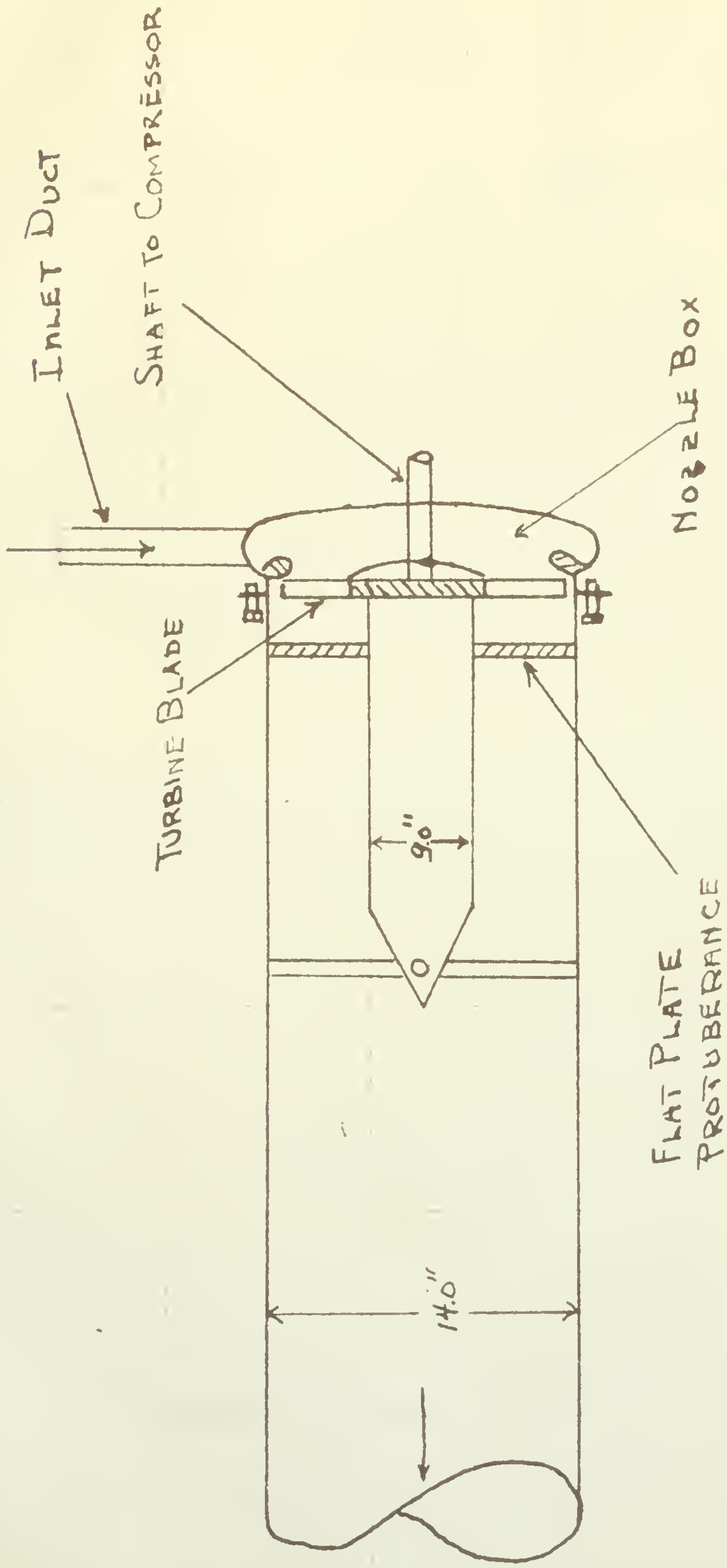
## -FIGURE I-

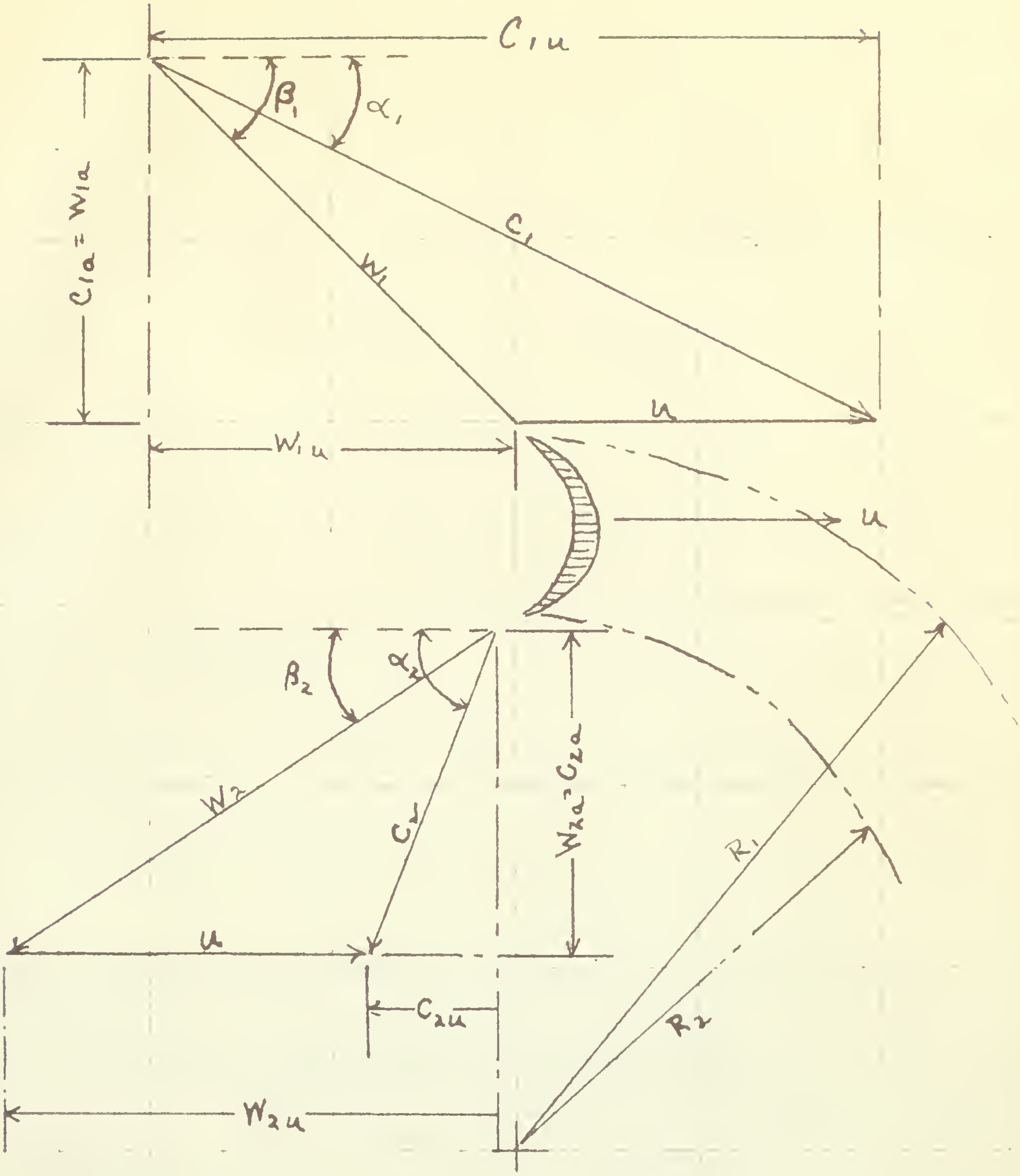
## SCHEMATIC DRAWING OF TEST EQUIPMENT



# - FIGURE II -

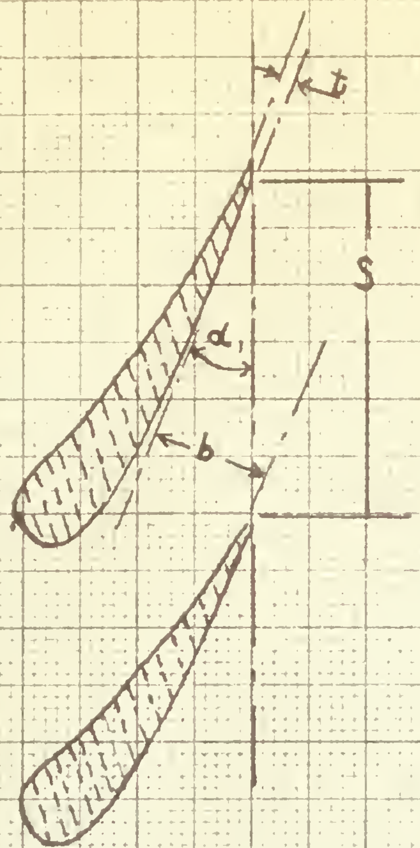
SIDE VIEW OF TURBINE AND EXIT ANNULUS
WITH FLAT PLATE PROTUBERANCES INSTALLED

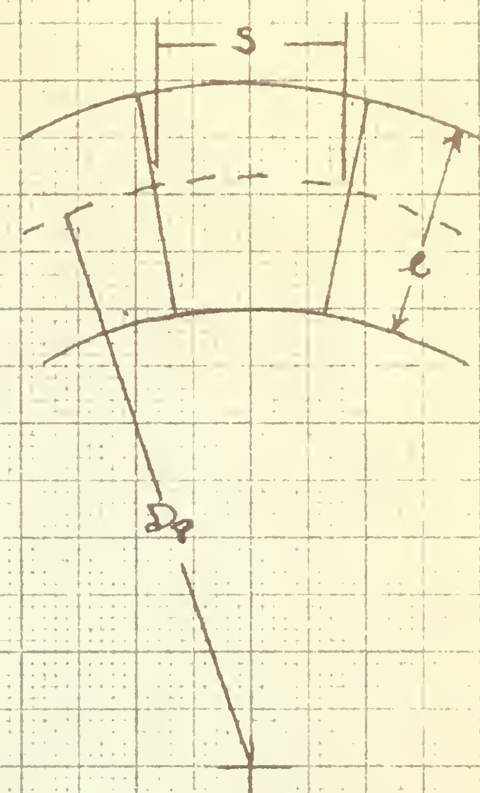





-FIGURE 3-

ENTRANCE AND EXIT VELOCITY DIAGRAMS FOR A MOVING BLADE PASSAGE IN A RATEAU STAGE




The effective flow area for one nozzle passage is in general:

$$A = bl$$

Then $\quad A = bl = (S \sin\alpha_1 - t)\,l$

OR, $\quad \sin\alpha_1 = \dfrac{bl + tl}{Sl} = \dfrac{b+t}{S}$

**Actual Dimensions**

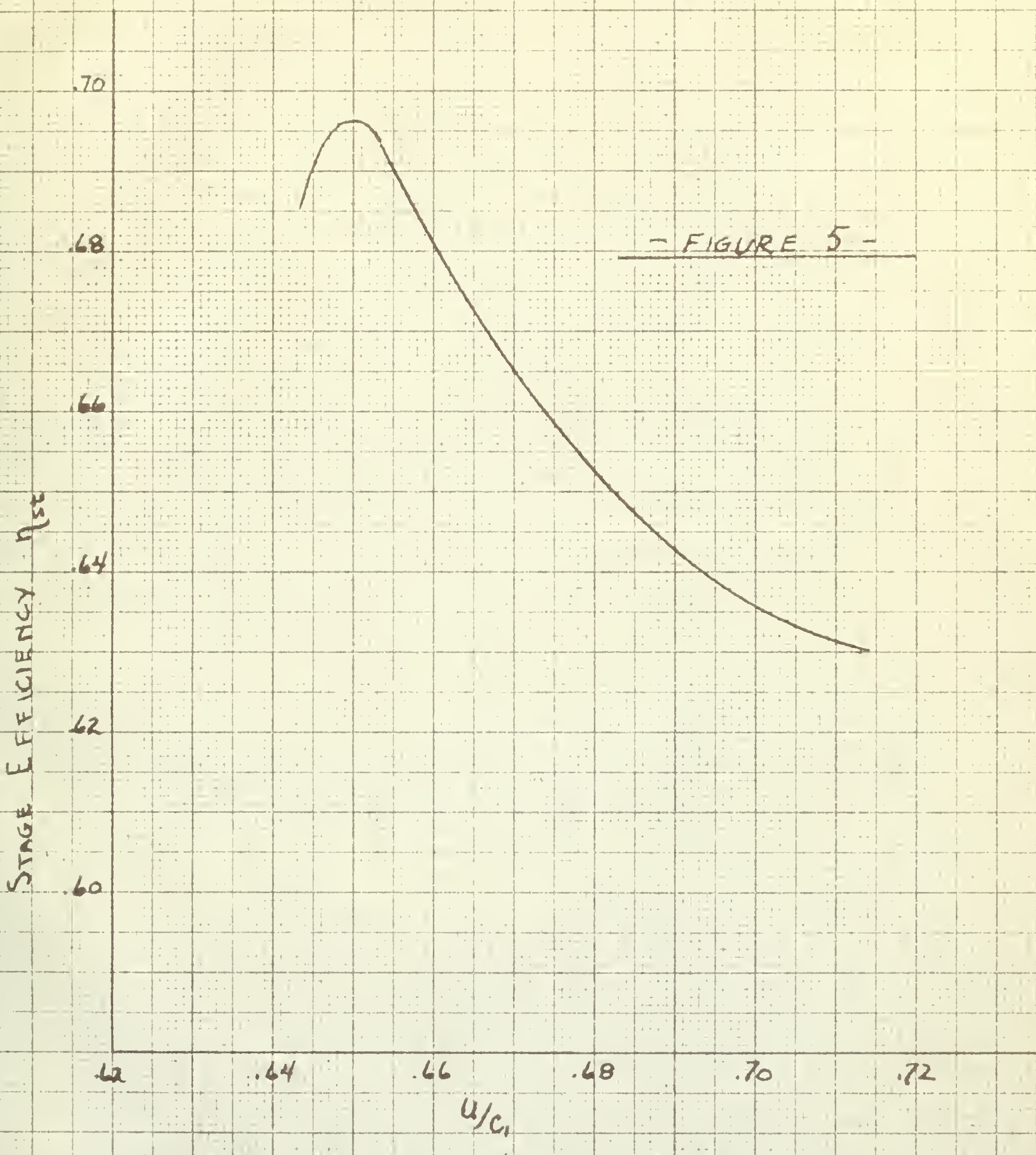
$S = .780$ in
$b = .200$ in
$t = .053$ in.

$\therefore \quad \alpha_1 = \sin^{-1} \dfrac{.253}{.780}$

$\alpha_1 = 18° 54'$

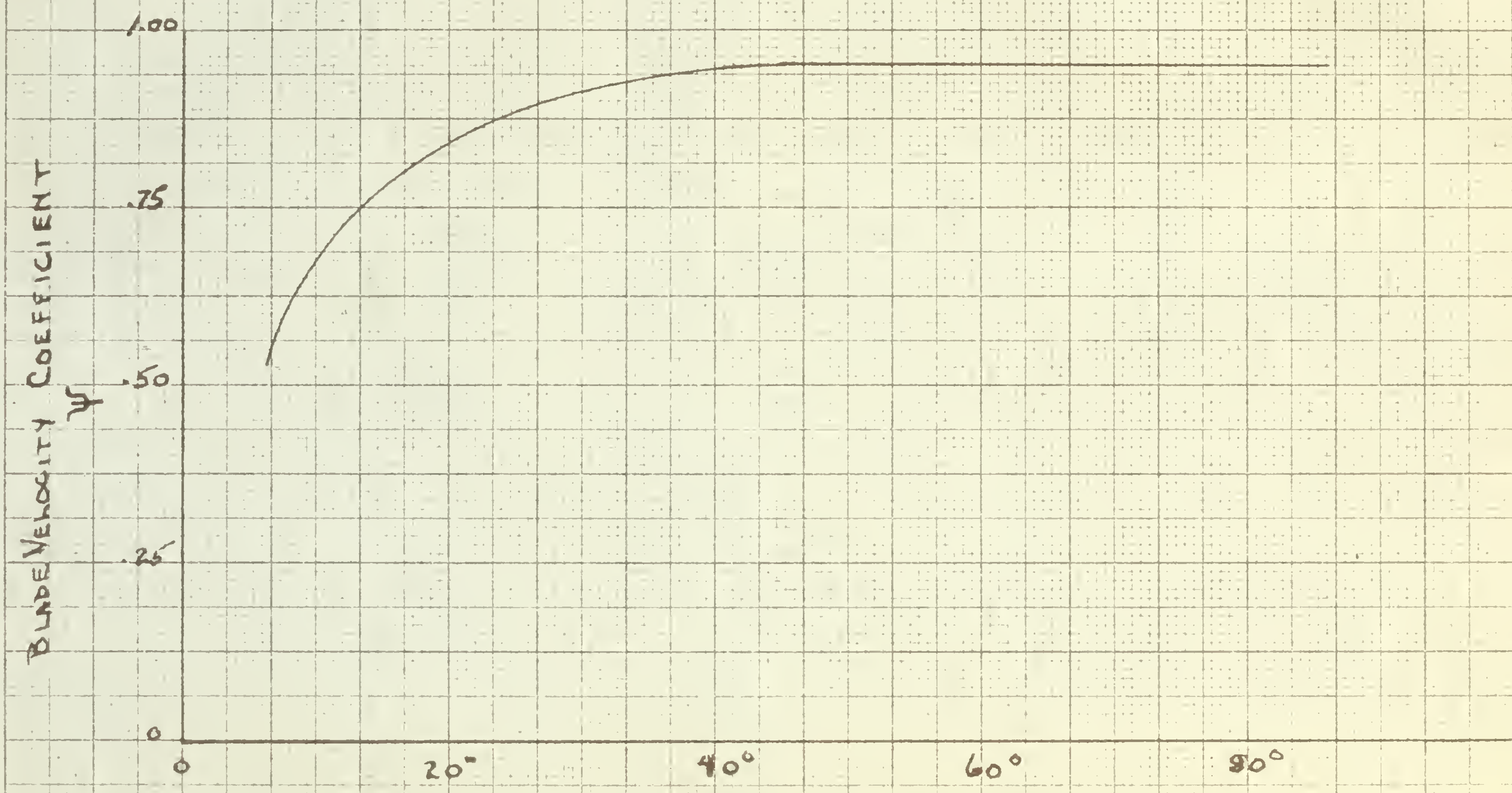
DETERMINATION OF NOZZLE EXIT ANGLE

- FIGURE 4 -

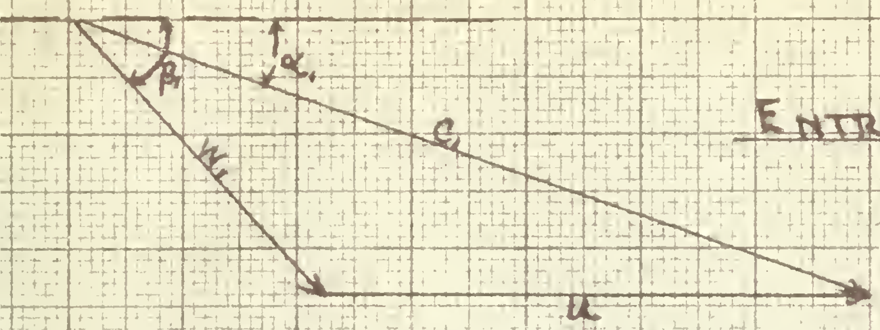



- FIGURE 5 -

TURBINE STAGE EFFICIENCY
-VS-
VELOCITY RATIO


1.00
.75
.50
.25
0
BLADE VELOCITY COEFFICIENT $\psi$
0
20°
40°
60°
80°
BLADE ANGLE

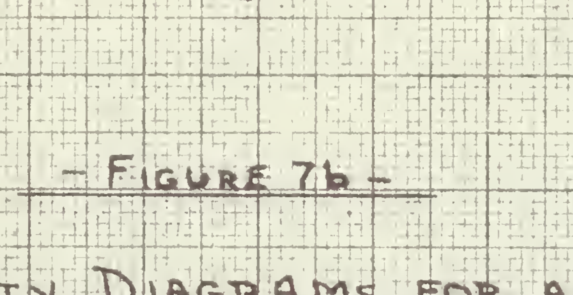
- FIGURE 6 -




ENTRANCE VELOCITY

Scale - 1 in = 200 ft./sec

\- FIGURE 7a -



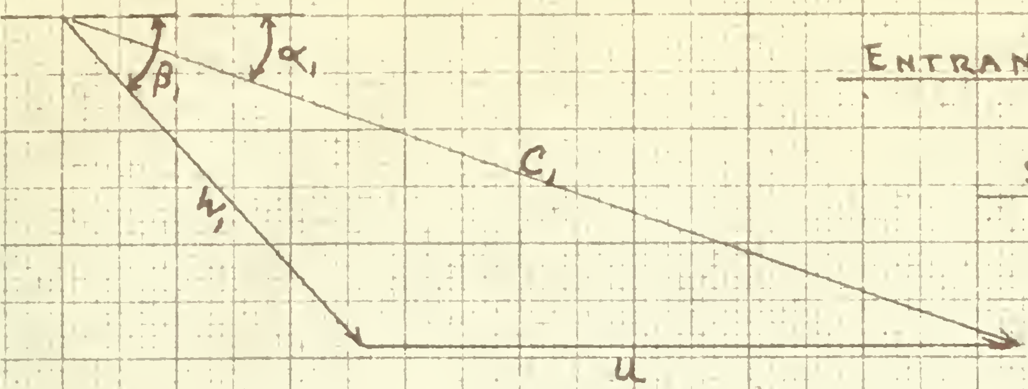

EXIT VELOCITY

Scale : 1 in = 200 ft/sec.

\- FIGURE 7b -

VELOCITY DIAGRAMS FOR A ROTOR SPEED

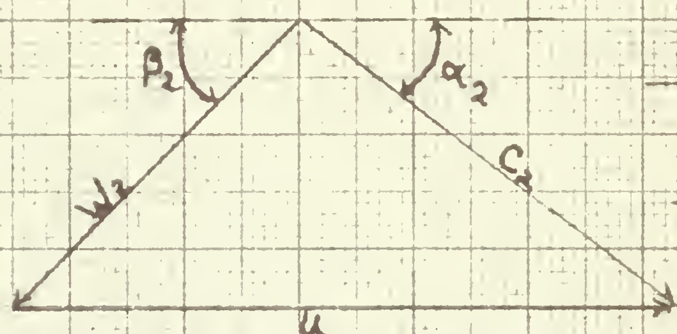
OF 10 000 RPM




ENTRANCE VELOCITY

Scale: 1 in. = 200 ft/sec

- FIGURE 8a -



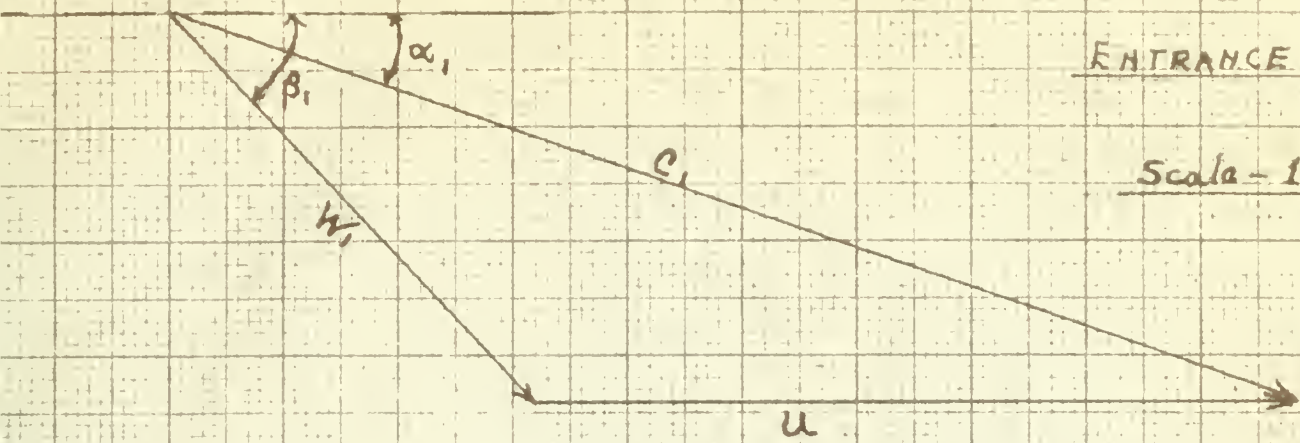

EXIT VELOCITY

Scale - 1 in. = 200 ft/sec

- FIGURE 8b -

VELOCITY DIAGRAMS FOR A ROTOR SPEED

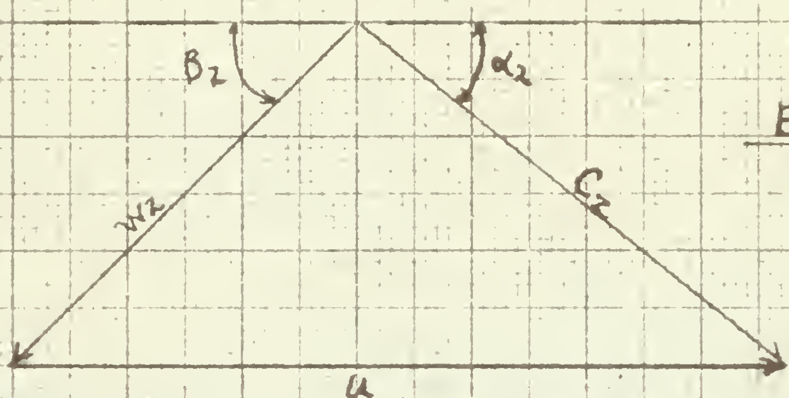
OF 12000 RPM




ENTRANCE VELOCITY

Scale - 1 in = 200 ft/sec

- FIGURE 9a -



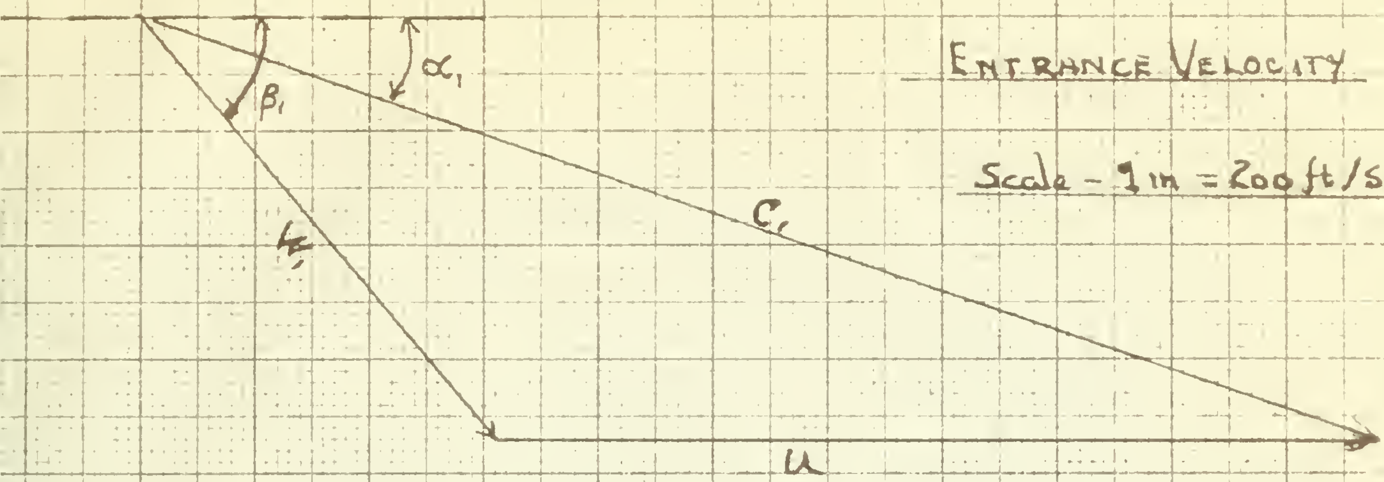

EXIT VELOCITY

Scale 1 in. = 200 ft/sec.

- FIGURE 9b -

VELOCITY DIAGRAMS FOR A ROTOR SPEED

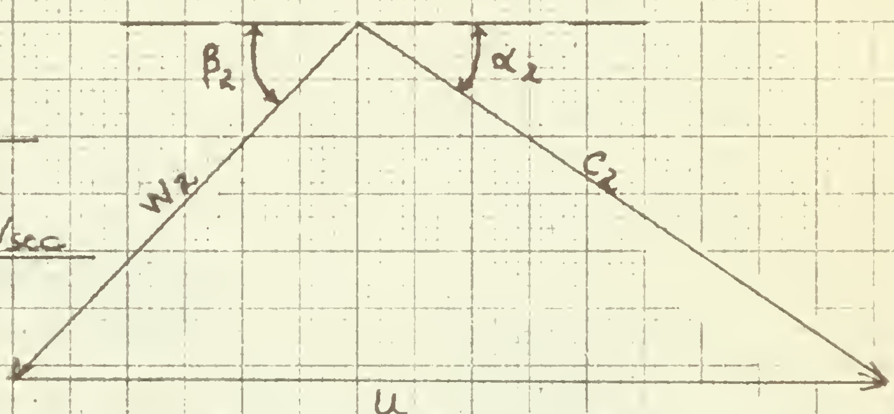
OF 14000 RPM




ENTRANCE VELOCITY

Scale - 1 in = 200 ft/sec

- FIGURE 10a -



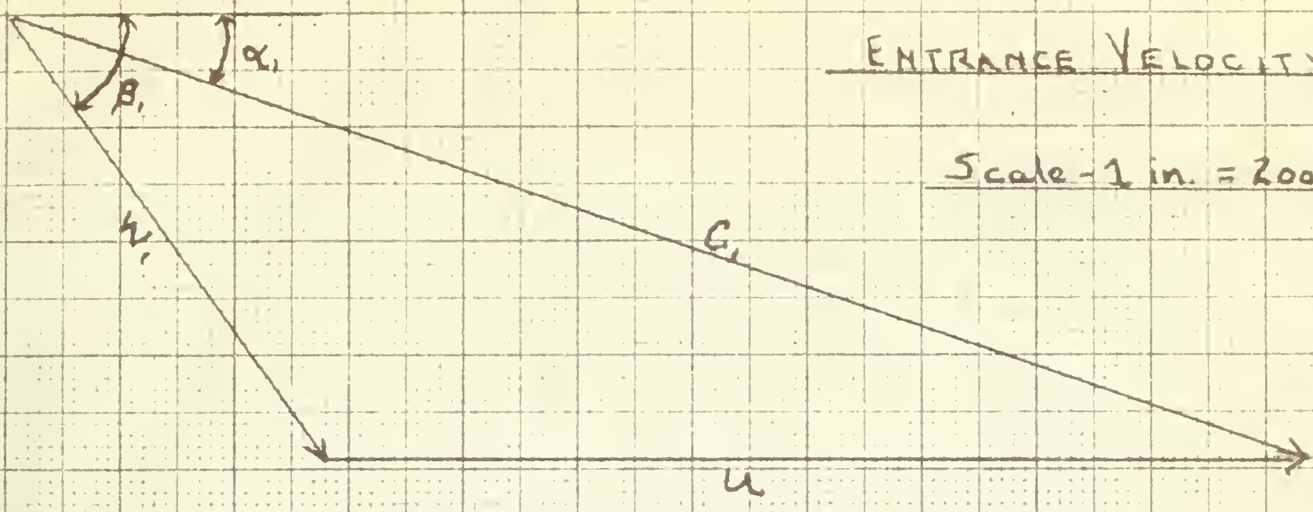

EXIT VELOCITY

Scale - 1 in = 200 ft/sec

- FIGURE 10b -

VELOCITY DIAGRAMS FOR A ROTOR SPEED

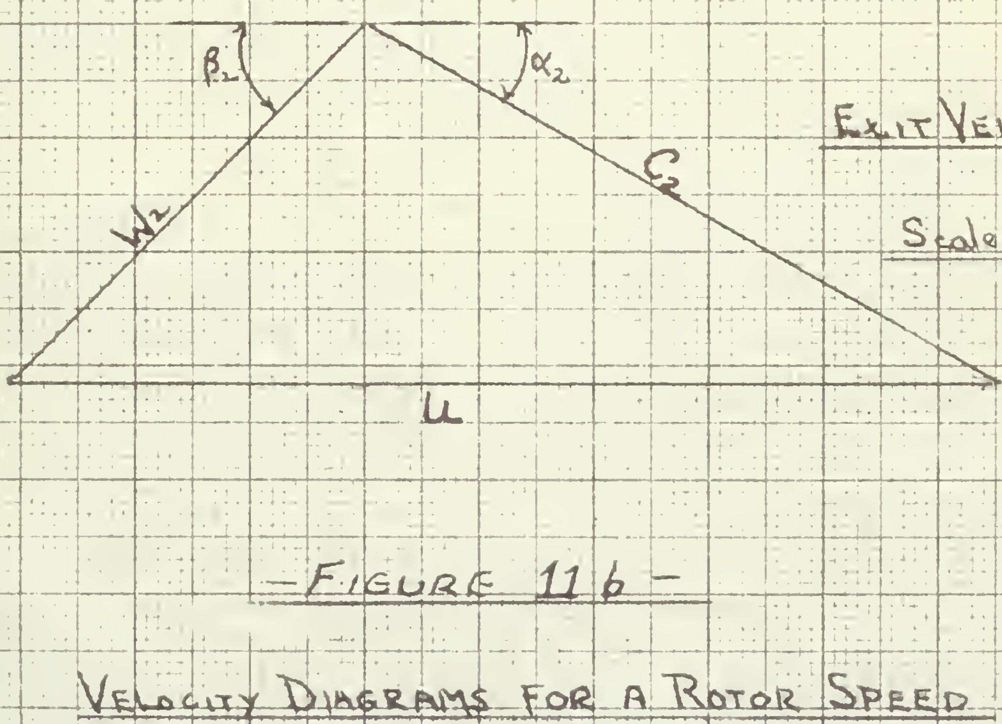
OF 16 000 RPM




ENTRANCE VELOCITY

Scale - 1 in. = 200 ft/sec.

- FIGURE 11 a -



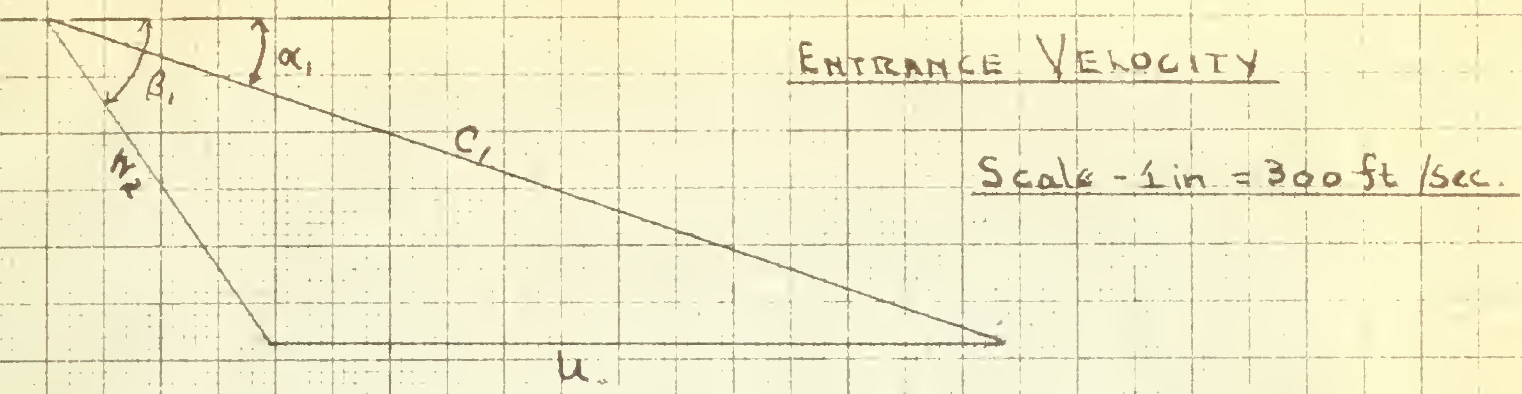

EXIT VELOCITY

Scale - 1 in = 200 ft./sec.

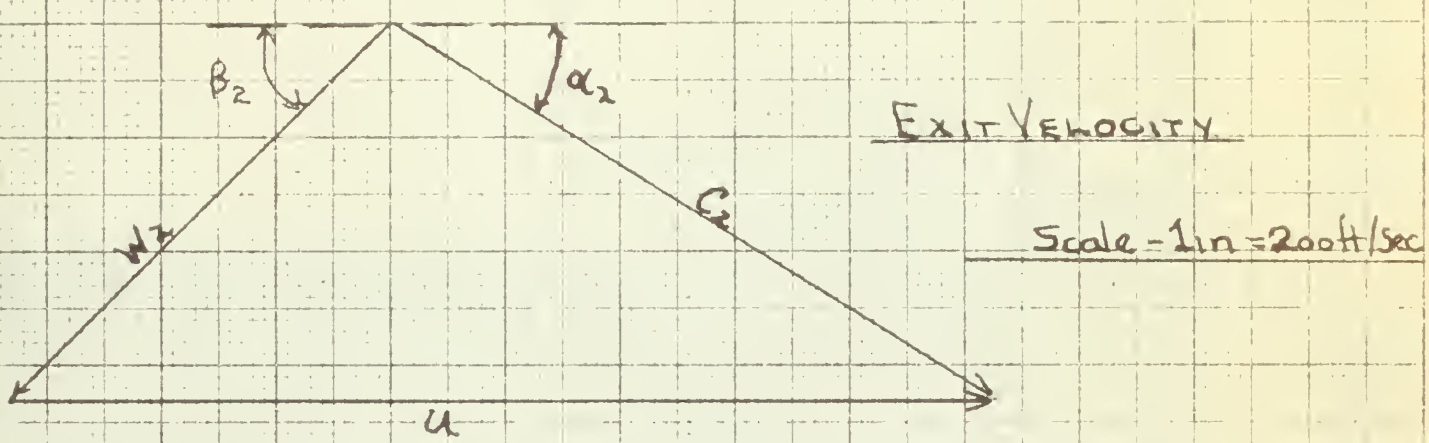
- FIGURE 11 b -

VELOCITY DIAGRAMS FOR A ROTOR SPEED

OF 18000 RPM



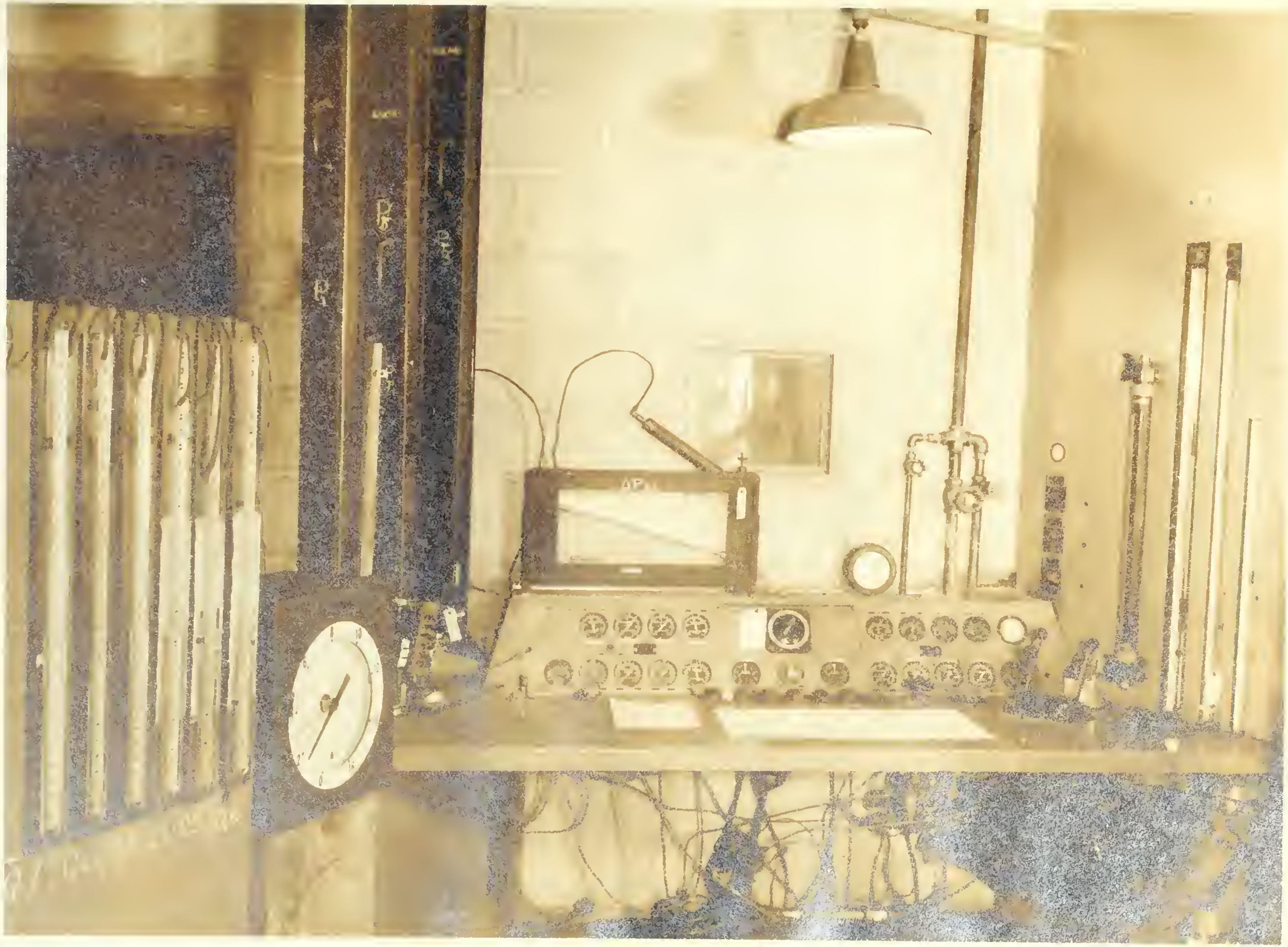

- FIGURE 12 a -



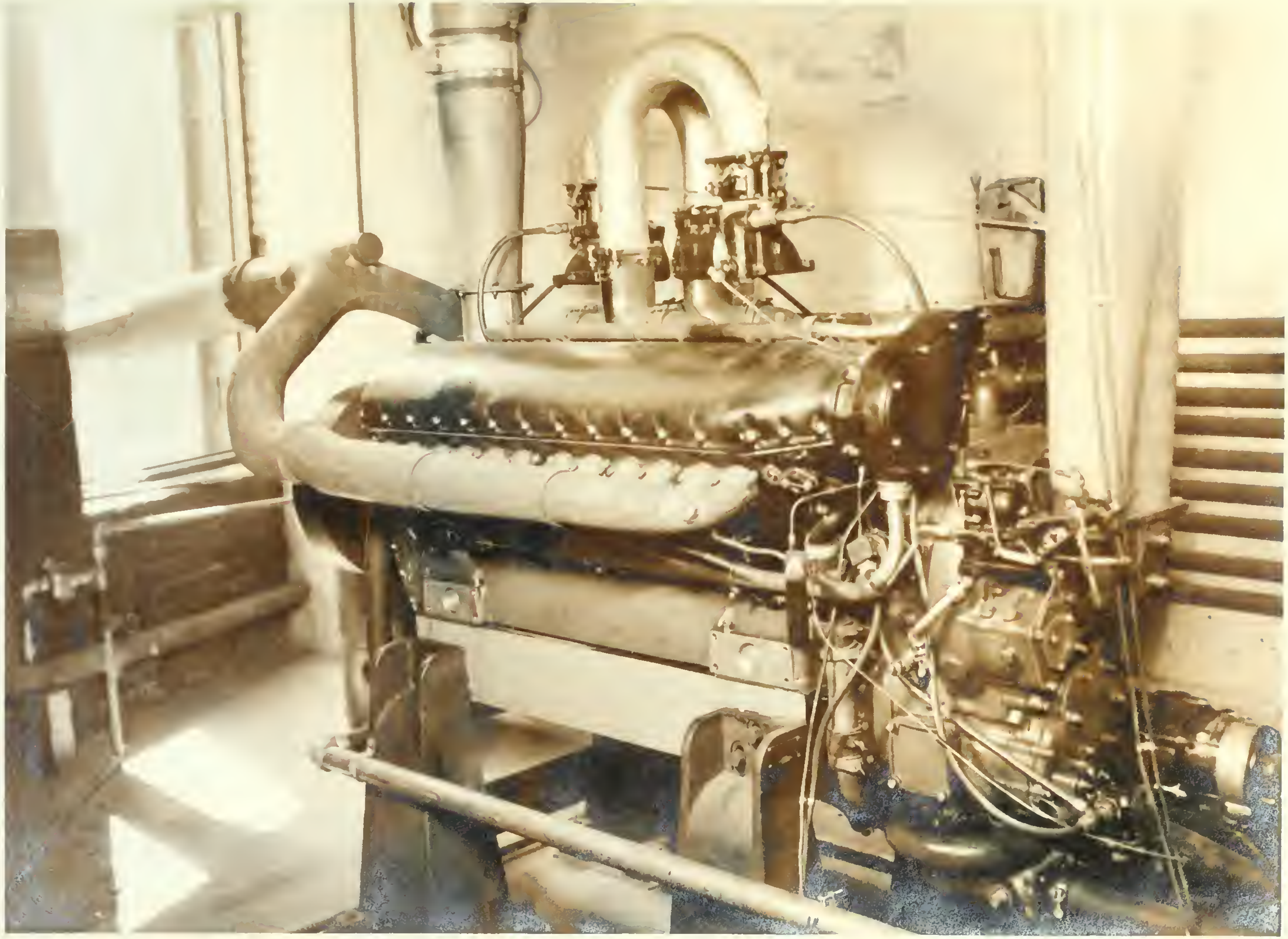

- FIGURE 12 b -

VELOCITY DIAGRAMS FOR A ROTOR SPEED

OF 20000 RPM



EXPERIMENTAL TEST PANEL



Allison Engine - Burner Air Supply

PROTUBERANCES IN Tailpipe ANNULUS

Thesis 11477
P95 Putnam
Impulse turbine flow analysis and correlation with experimental data turbine efficiency variation with protuburances in tail-pipe.